Available online at www.sciencedirect.com
ScienceDirect
International Journal of Heat and Mass Transfer 50 (2007) 5318–5322
International Journal of HEAT and MASS TRANSFER
www.elsevier.com/locate/ijhmt

Technical Note

Closed-form correlations for thermal optimization of microchannels

Dong-Kwon Kim, Sung Jin Kim *

Department of Mechanical Engineering, Korea Advanced Institute of Science and Technology, Taejon 305-701, Republic of Korea

Received 19 December 2006; received in revised form 24 July 2007
Available online 24 October 2007

Abstract

In the present paper, closed-form correlations which allow for thermal optimization of microchannels are suggested. Asymptotic solutions for velocity and temperature distributions for the high channel-aspect-ratios, high conductivity ratios, and low Reynolds numbers are presented using the averaging approach. From the asymptotic solutions, explicit correlations for the optimal channel width and the optimal fin thickness are proposed.
© 2007 Elsevier Ltd. All rights reserved.

1. Introduction

The concept of the microchannels for cooling high power electronic devices was first introduced by Tuckerman and Pease [1]. It is based on the fact that the heat transfer coefficient is inversely proportional to the hydraulic diameter of the channel. Since the emergence of this cooling technology, much research has been conducted.

Many investigations have focused on optimum design methodologies which determine the dimensions such as the channel width and the fin thickness for the optimal performance of microchannels. However, there is no practical closed-form correlation by which one can obtain the optimum geometry of the microchannels directly. Knight et al. [2] presented an optimization method based on a fin model. Their method is commonly used, but it has been recently confirmed that their method fails to provide the optimum geometry unless the fin efficiency is arbitrarily guessed when the bottom wall is uniformly heated and the pumping power is constant [3]. Bejan and Sciubba [4] also suggested an optimization method for determining the optimal spacing of stacked parallel plates. However, this method does not provide any information about the optimal fin thickness. Ryu et al. [5] conducted full three-dimensional numerical simulation to optimize the thermal performance of the microchannels. Direct three-dimensional numerical simulation estimates the optimal dimensions accurately but requires a time-consuming and iterative process. Kim [3] reported an optimization method using the averaging approach. The microchannels were modeled as a fluid-saturated porous medium and the averaged velocity and temperature distributions were analytically obtained to describe fluid flow and heat transfer phenomena in the microchannels. However, when the model based on the averaging approach is used, optimization work needs to be done iteratively due to their implicit nature of solutions [6]. Therefore, it is necessary to suggest closed-form correlations which allow for thermal optimization of microchannels with ease.

The purpose of present study is to present closed-form correlations which can be used for optimizing the thermal performance of the microchannels in the early stage of thermal design. In order to present closed-form correlations, asymptotic solutions for velocity and temperature distributions for the microchannels with high channel-aspect-ratios are obtained using the averaging approach. The solutions are validated by comparing them with the results of direct numerical simulation and the experimental data. From the asymptotic solutions, explicit correlations for the optimal channel width and the optimal fin thickness are proposed. The characteristics and limitations of the suggested correlations are discussed.

* Corresponding author. Tel.: +82 42 869 3083; fax: +82 42 869 8207.
E-mail address: sungjinkim@kaisf.ac.kr (S.J. Kim).

0017-9310/$ - see front matter © 2007 Elsevier Ltd. All rights reserved.
doi:10.1016/j.ijheatmasstransfer.2007.07.034

Nomenclature

a	wetted area per volume
h	heat transfer coefficient based on one-dimensional bulk mean temperature
h_{I}	interstitial heat transfer coefficient
K	permeability
P_{pump}	pumping power
q	heat transfer rate
q''	heat flux
q''_{sf}	heat flux from the fins to the fluid
T	temperature
u	velocity
w_{c}	channel width
w_{w}	fin thickness

Greek symbols

α	aspect ratio of the channel (H/w_{c})
ε	porosity ($w_{\mathrm{c}}/(w_{\mathrm{w}}+w_{\mathrm{c}})$)

Subscripts

f	fluid
s	solid

2. Mathematical formulation

The problem under consideration in the present paper concerns forced convection through microchannels as depicted in Fig. 1. The direction of fluid flow is parallel to x direction. The top surface is insulated and the bottom surface is uniformly heated. A coolant passes through a number of microchannels and takes heat away from a heat-dissipating electronic component attached below. In analyzing the problem, for simplicity, the flow is assumed to be laminar, incompressible, and both hydrodynamically and thermally fully-developed. All thermophysical properties are assumed to be constant. In addition, pumping power is assumed to be constant. This condition means that the power required to drive the fluid through the microchannels is fixed. It is also assumed that the aspect ratio of the channel is much higher than 1 and the solid conductivity is higher than the fluid conductivity.

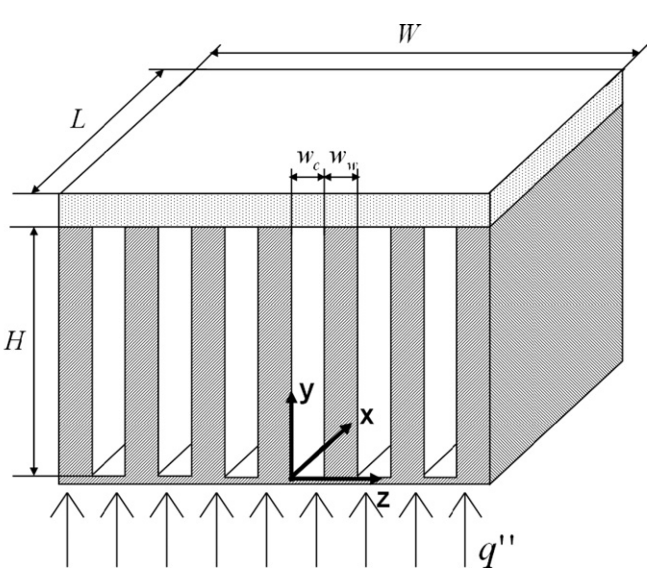

Fig. 1. Schematic diagram of microchannels.

In the averaging approach, averaged velocity and temperature are used. The averaged quantities over the fluid and solid phases are defined, respectively, as follows:

$$\langle\phi\rangle^{\mathrm{f}} = \frac{1}{w_{\mathrm{c}}}\int_0^{w_{\mathrm{c}}} \phi\,\mathrm{d}z, \quad \langle\phi\rangle^{\mathrm{s}} = \frac{1}{w_{\mathrm{w}}}\int_{w_{\mathrm{c}}}^{w_{\mathrm{w}}+w_{\mathrm{c}}} \phi\,\mathrm{d}z \tag{1}$$

The governing equations for averaged velocity and temperature are established by averaging the momentum and energy equations in the z direction [6]. When $H \gg w_{\mathrm{c}}$, $k_{\mathrm{s}} \gg k_{\mathrm{f}}$, the governing equations are given as follows:

$$\frac{\mathrm{d}\langle p\rangle^{\mathrm{f}}}{\mathrm{d}x} = -\frac{\mu_{\mathrm{f}}}{K}\varepsilon\langle u\rangle^{\mathrm{f}} \tag{2}$$

$$\varepsilon\rho_{\mathrm{f}}c_{\mathrm{f}}\langle u\rangle^{\mathrm{f}}\frac{\partial\langle T\rangle^{\mathrm{f}}}{\partial x} = h_{\mathrm{I}}a(\langle T\rangle^{\mathrm{s}} - \langle T\rangle^{\mathrm{f}}) \tag{3}$$

$$\frac{\partial}{\partial y}\left(k_{\mathrm{se}}\frac{\partial\langle T\rangle^{\mathrm{s}}}{\partial y}\right) = h_{\mathrm{I}}a(\langle T\rangle^{\mathrm{s}} - \langle T\rangle^{\mathrm{f}}) \tag{4}$$

where ε, a, k_{fe}, k_{se}, K and h_{I} are porosity, wetted area per volume, effective conductivity of the fluid, effective conductivity of the solid, permeability, and interstitial heat transfer coefficient, respectively:

$$\varepsilon = \frac{w_{\mathrm{c}}}{w_{\mathrm{c}}+w_{\mathrm{w}}}, \quad a = \frac{2}{w_{\mathrm{w}}+w_{\mathrm{c}}}, \quad k_{\mathrm{se}} = (1-\varepsilon)k_{\mathrm{s}}, \quad k_{\mathrm{fe}} = \varepsilon k_{\mathrm{f}} \tag{5}$$

$$K \equiv -\varepsilon w_{\mathrm{c}}\langle u\rangle^{\mathrm{f}}\left(\left.\frac{\partial u}{\partial z}\right|_{z=w_{\mathrm{c}}} - \left.\frac{\partial u}{\partial z}\right|_{z=0}\right)^{-1} \tag{6}$$

$$h_{\mathrm{I}} \equiv \frac{q''_{\mathrm{sf}}}{\langle T\rangle^{\mathrm{s}} - \langle T\rangle^{\mathrm{f}}} = \frac{k_{\mathrm{f}}}{2}\left(\left.\frac{\partial T}{\partial z}\right|_{z=w_{\mathrm{c}}} - \left.\frac{\partial T}{\partial z}\right|_{z=0}\right)\cdot\left(\langle T\rangle^{\mathrm{s}} - \langle T\rangle^{\mathrm{f}}\right)^{-1} \tag{7}$$

The boundary conditions are given as follows [6]:

$$\langle T\rangle^{\mathrm{s}} = T_{\mathrm{w}} \quad \text{at } y=0, \quad \frac{\partial\langle T\rangle^{\mathrm{s}}}{\partial y} = 0 \quad \text{at } y=H \tag{8}$$

To solve the governing equations, (2)–(4), the permeability K and the interstitial heat transfer coefficient h_{I} should be determined in advance. For this, it is assumed that the characteristics of pressure drop and heat transfer from the fins are similar to those of the Poiseuille flow between two infinite parallel plates. The velocity and temperature distributions for the Poiseuille flow in this configuration can be easily determined to be

$$u = 6\langle u\rangle^{\mathrm{f}} \cdot \frac{z}{w_{\mathrm{c}}}\left(1 - \frac{z}{w_{\mathrm{c}}}\right),$$
$$T = 30(\langle T\rangle^{\mathrm{b,f}} - \langle T\rangle^{\mathrm{s}})\left(\frac{1}{6}\left(\frac{z}{w_{\mathrm{c}}}\right)^4 - \frac{1}{3}\left(\frac{z}{w_{\mathrm{c}}}\right)^3 + \frac{1}{6}\left(\frac{z}{w_{\mathrm{c}}}\right)\right) + \langle T\rangle^{\mathrm{s}} \tag{9}$$

Using Eq. (9), the permeability and the interstitial heat transfer coefficient are obtained from their definitions, Eqs. (6) and (7), as

$$K = \frac{\varepsilon w_{\mathrm{c}}^2}{12}, \quad h_{\mathrm{l}} = \frac{5k_{\mathrm{f}}}{w_{\mathrm{c}}} \tag{10}$$

Once the permeability and the interstitial heat transfer coefficient are determined, velocity and temperature distributions can be obtained by solving Eqs. (2)–(4). Finally, the asymptotic solutions are given as follows:

$$U^{\mathrm{f}} = -P = 1 \tag{11}$$

$$\theta^{\mathrm{s}} = \frac{1}{2}(Y^2 - 2Y) \tag{12}$$

$$\theta^{\mathrm{f}} = \frac{1}{2}(Y^2 - 2Y) - \frac{(1-\varepsilon)k_{\mathrm{s}}}{2\varepsilon\alpha h_{\mathrm{l}} H} \tag{13}$$

where

$$\alpha = \frac{H}{w_{\mathrm{c}}}, \quad Y = \frac{y}{H}, \quad U^{\mathrm{f}} = \frac{\langle u\rangle^{\mathrm{f}}}{u_{\mathrm{m}}}, \quad P = \frac{K}{\varepsilon\mu_{\mathrm{f}} u_{\mathrm{m}}}\frac{\mathrm{d}\langle p\rangle^{\mathrm{f}}}{\mathrm{d}x},$$
$$\theta^{\mathrm{s}} = \frac{\langle T\rangle^{\mathrm{s}} - T_{\mathrm{w}}}{\frac{q''H}{(1-\varepsilon)k_{\mathrm{s}}}}, \quad \theta^{\mathrm{f}} = \frac{\langle T\rangle^{\mathrm{f}} - T_{\mathrm{w}}}{\frac{q''H}{(1-\varepsilon)k_{\mathrm{s}}}} \tag{14}$$

In order to calculate the bulk mean temperature of the fluid, the one-dimensional bulk mean temperature for the fluid is defined as the following equation:

$$\langle T\rangle^{\mathrm{f,b}} = \frac{\int_0^{w_{\mathrm{c}}} Tu\,\mathrm{d}z}{\int_0^{w_{\mathrm{c}}} u\,\mathrm{d}z} \tag{15}$$

This one-dimensional bulk mean temperature satisfies the following equation:

$$h = \frac{q''_{\mathrm{sf}}}{\langle T\rangle^{\mathrm{s}} - \langle T\rangle^{\mathrm{f,b}}} = \frac{70}{17}\frac{k_{\mathrm{f}}}{w_{\mathrm{c}}} \tag{16}$$

Combining Eqs. (7), (10) and (16),

$$\theta^{\mathrm{f,b}} \equiv \frac{\langle T\rangle^{\mathrm{f,b}} - T_{\mathrm{w}}}{\frac{q''H}{(1-\varepsilon)k_{\mathrm{s}}}} = \theta^{\mathrm{s}} - \frac{17}{14}(\theta^{\mathrm{s}} - \theta^{\mathrm{f}}) \tag{17}$$

The bulk mean temperature of the fluid is given as

$$T_{\mathrm{bm}} = \frac{\int_0^H \int_0^{w_{\mathrm{c}}} Tu\,\mathrm{d}z\,\mathrm{d}y}{\int_0^H \int_0^{w_{\mathrm{c}}} u\,\mathrm{d}z\,\mathrm{d}y} \simeq \frac{q''H}{(1-\varepsilon)k_{\mathrm{s}}}\int_0^1 \theta^{\mathrm{f,b}}\,\mathrm{d}Y + T_{\mathrm{w}} \tag{18}$$

3. Results and discussion

The concepts of the friction factor and the Nusselt number are typically used for describing macroscopic quantities such as the average pressure drop across a channel and the average heat transfer rate from a wall. Combining to Eqs. (10), (11) and (14), the friction factor for high channel-aspect-ratios is expressed as

$$fRe_{D_{\mathrm{h}}} = 24\left(\frac{\alpha}{\alpha+1}\right)^2 \tag{19}$$

According to Eqs. (12), (13), (17) and (18), the Nusselt number for high channel-aspect-ratios is given as

$$Nu_{\mathrm{H}} \equiv \frac{q''}{(T_{\mathrm{w}} - T_{\mathrm{bm}})}\frac{H}{k_{\mathrm{f}}} = \frac{1}{3(1-\varepsilon)}\left(\frac{k_{\mathrm{f}}}{k_{\mathrm{s}}}\right) + \frac{17}{140\varepsilon\alpha^2} \tag{20}$$

The thermal performance of the microchannels can be evaluated by introducing the concept of the thermal resistance. The total thermal resistance is defined as the temperature difference between the base temperature of the microchannels at the outlet and the fluid bulk mean temperature at the inlet per unit heat flow rate.

$$R_{\theta,\mathrm{tot}} = \frac{T_{\mathrm{w,out}} - T_{\mathrm{bm,in}}}{q} \tag{21}$$

The asymptotic solutions for the velocity and temperature distributions yield

$$R_{\theta,\mathrm{tot}} = \frac{1}{3}\frac{(w_{\mathrm{c}} + w_{\mathrm{w}})H}{k_{\mathrm{s}} w_{\mathrm{w}} WL} + \frac{17}{140}\frac{w_{\mathrm{c}}(w_{\mathrm{c}} + w_{\mathrm{w}})}{k_{\mathrm{f}} HWL} + \frac{\sqrt{12\mu_{\mathrm{f}}(w_{\mathrm{c}} + w_{\mathrm{w}})L}}{\rho_{\mathrm{f}} c_{\mathrm{f}}\sqrt{w_{\mathrm{c}}^3 WHP_{\mathrm{pump}}}} \tag{22}$$

where $\dot{Q}$ and P_{pump} are the flow rate and the pumping power, respectively. The pumping power is defined as

$$P_{\mathrm{pump}} = \dot{Q} \cdot \Delta p \tag{23}$$

In order to validate the solutions presented in the previous section, thermal resistances from asymptotic solutions are compared with results of the two-dimensional direct numerical simulation. Numerical solutions are obtained by solving the momentum equation and energy equation using the control-volume-base finite difference method. Fig. 2 depicts the thermal resistances for various channel aspect ratios which are calculated using the asymptotic solutions and the numerical simulation. The thermal resistances obtained from the asymptotic solutions match with those from numerical results within a relative error of 10% when the channel aspect ratio is higher than 4.

In addition to the validation using the numerical results, the present results are also compared with experimental data by Qu and Mudawar [7]. Fig. 3 presents the friction factors and the thermal resistances. As shown in Fig. 3, the results obtained from the asymptotic solutions predict the experimental data moderately well for $Re < 600$. However it is shown that the friction factor obtained from the asymptotic solution deviates from experimental value as

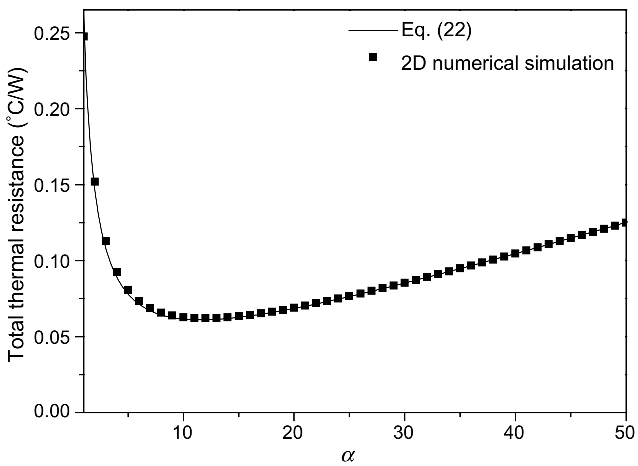

Fig. 2. Total thermal resistances ($w_c = w_w = 50$ μm, $L = W = 1$ cm, $\mu_f = 0.000855$ kg/m s, $c_f = 4179$ J/kg K, $\rho_f = 997$ kg/m^3, $k_s = 148$ W/m K, $k_f = 0.613$ W/m K, $P_{pump} = 2.56$ W).

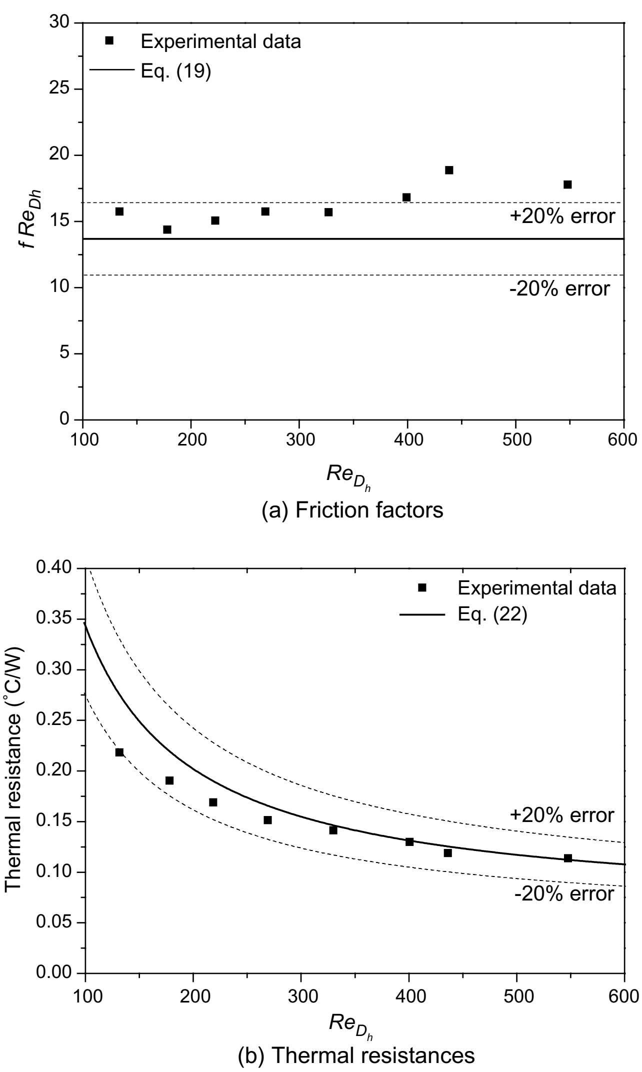

Fig. 3. Comparison between results from the model and experimental data.

the Reynolds number increases. It is due to the limitations of the current model. The limitations come from the assumptions adopted for simplifying the problem and eliminating implicit nature of the solutions. Firstly, the flow is assumed to be both hydrodynamically and thermally fully-developed. This assumption neglects entry region effects and leads to accurate prediction only if $L/D_h > 0.05\,Re_{D_h}$ and $L/D_h > 0.05\,Re_{D_h} Pr$. When the fluid is water and $L/D_h = 200$, the present model is valid for $Re_{D_h} < 690$. In the case of air, the model is applicable for $Re_{D_h} < 2000$. Secondly, it is assumed that the aspect ratio of the channel is higher than 1 and the solid conductivity is higher than the fluid conductivity. From these assumptions, the valid range is limited to $k_s/k_f > 20$ and $H/w_c > 4$. Thirdly, the inlet and exit plenums are not considered in this work. Therefore the present model is not proper when the pressure loss at the inlet and exit is comparable to the friction loss in the microchannels. In short, the proposed correlations are valid when the channel aspect ratio is high ($H/w_c > 4$), the conductivity ratio is high ($k_s/k_f > 20$), and the Reynolds number is low ($Re_{D_h} < 690$).

To design the microchannels, the important dimensions such as the fin thickness and the channel width for which the total thermal resistance is minimized should be determined. The optimal fin thickness and channel width for the microchannels are determined by

$$\frac{\partial R_{\theta,\text{tot}}}{\partial w_c} = \frac{\partial R_{\theta,\text{tot}}}{\partial w_w} = 0 \tag{24}$$

From Eqs. (22) and (24), closed-form correlations for the optimal fin thickness and channel width are given as follows:

$$w_w = H\sqrt{\frac{70k_f}{51k_s}} \tag{25}$$

$$w_c^6 + H\sqrt{\frac{70k_f}{51k_s}}w_c^5 = \frac{3\cdot(140)^2 k_f^2 \mu_f W L^3 H}{(17)^2 \rho_f^2 c_f^2 P_{pump}} \tag{26}$$

Fig. 4 shows the optimized design variables for various heights. Suggested correlations evaluate the optimum geometry within a relative error of 10% compared to results of two-dimensional direct numerical simulation.

By using the proposed correlations, we can investigate the characteristics of the optimal fin thickness and channel width for the high channel-aspect-ratios, high conductivity ratios, and low Reynolds numbers:

(1) The optimal fin thickness is shown to depend on the height, the solid conductivity, and the fluid conductivity only. Surprisingly, the optimal fin thickness is independent of the pumping power, the viscosity of the fluid, and the length of the microchannels.
(2) Another interesting fact implied in Eq. (25) is that the optimal fin thickness is linearly proportional to the height.
(3) Generally, the optimal channel width is given as a complicated function of the fluid properties, the solid

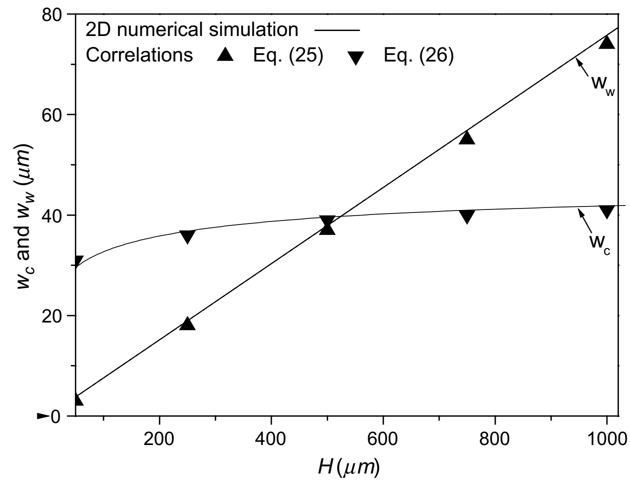

Fig. 4. Optimized design variables ($L = W = 1$ cm, $\mu_f = 0.000855$ kg/ms, $c_f = 4179$ J/kg K, $\rho_f = 997$ kg/m^3, $k_s = 148$ W/m K, $k_f = 0.613$ W/m K, $P_{pump} = 2.56$ W).

properties, the total size of the microchannels, and the pumping power.

(4) For special cases, the optimal channel width can be more simply expressed as follows:

$$w_c = \left(\frac{3\cdot(140)^2 k_f^2 \mu_f W L^3 H}{(17)^2 \rho_f^2 c_f^2 P_{pump}}\right)^{\frac{1}{6}}$$

$$\text{for } \frac{3\cdot(140)^2 k_f^2 \mu_f W L^3 H}{(17)^2 \rho_f^2 c_f^2 P_{pump}}\left(H\sqrt{\frac{70k_f}{51k_s}}\right)^{-6} \gg 1 \qquad (27)$$

$$w_c = \left(\frac{3\cdot(140)^2 k_f^2 \mu_f W L^3 H}{(17)^2 \rho_f^2 c_f^2 P_{pump}}\right)^{\frac{1}{5}}\left(H\sqrt{\frac{70k_f}{51k_s}}\right)^{-\frac{1}{5}}$$

$$\text{for } \frac{3\cdot(140)^2 k_f^2 \mu_f W L^3 H}{(17)^2 \rho_f^2 c_f^2 P_{pump}}\left(H\sqrt{\frac{70k_f}{51k_s}}\right)^{-6} \ll 1 \qquad (28)$$

4. Conclusion

In the present study, a simple way to predict the optimal dimensions of the microchannels is suggested. New correlations for the optimal channel width and the optimal fin thickness are proposed. Even though the applicability of the suggested correlations are limited to the situation for the high channel-aspect-ratios ($H/w_c > 4$), high conductivity ratios ($k_s/k_f > 20$), and low Reynolds numbers ($Re_{D_h} < 690$), the proposed correlations provide insight into thermal design and allow for easier engineering optimization. The present correlations may be used for design of the microchannels in the early stage of thermal design.

Acknowledgements

This work was supported by the Korea Science and Engineering Foundation (KOSEF) through the National Research Lab. Program funded by the Ministry of Science and Technology (No. M1060000022406J000022410).

References

[1] D.B. Tuckerman, R.F.W. Pease, High-performance heat sinking for VLSI, IEEE Electr. Dev. Lett. 2 (1981) 126–129.
[2] R.W. Knight, J.S. Goodling, D.J. Hall, Optimal thermal design of forced convection heat sinks – analytical, J. Electron. Packaging 113 (1991) 313–321.
[3] S.J. Kim, Methods for thermal optimization of microchannel heat sinks, Heat Transfer Eng. 25 (1) (2004) 37–49.
[4] A. Bejan, E. Sciubba, The optimal spacing of parallel plates cooled by forced convection, Int. J. Heat Mass Transfer 35 (1992) 3259–3264.
[5] J.H. Ryu, D.H. Choi, S.J. Kim, Numerical optimization of the thermal performance of a microchannel heat sink, Int. J. Heat Mass Transfer 45 (2002) 2823–2827.
[6] S.J. Kim, D. Kim, D.Y. Lee, On the local thermal equilibrium in microchannel heat sinks, Int. J. Heat Mass Transfer 43 (2000) 1735–1748.
[7] W. Qu, I. Mudawar, Experimental and numerical study of pressure drop and heat transfer in a single-phase micro-channel heat sink, Int. J. Heat Mass Transfer 45 (2002) 2549–2565.